Tension Buckling Behaviour of Paper

Y.B. SEO, R. CHAVES DE OLIVEIRA and R.E. MARK

When thin rectangular specimens of materials such as paper are tested in tension, wrinkles parallel to the loading direction often appear. The condition is termed "tension buckling". The propagation of these wrinkles can be confined to small areas near the grips if necked-down specimens are used. The authors use finite element analysis and mechanical and optical test methods to demonstrate the phenomenon, its magnitude, and methods to control it in this paper.

INTRODUCTION

In paper testing, the uniaxial tensile test is the most important test for evaluation of paper mechanical properties. Not only can simple parameters such as tensile strength and Young's modulus be measured, but in-plane Poisson ratio [1], out-of-plane Poisson ratio, and in-plane shear modulus [2] can also be determined if the tests are conducted carefully and certain principles are understood. Futhermore, the mechanical inhomogeneity of the paper can be quantified [1].

Powerful optical techniques [1,3,4] for measuring paper deformation two-dimensionally have been recently developed to replace the simple displacement measurements performed by reading crosshead movement of the paper tensioning device. However, the accuracy of all these optical techniques largely depends on the way the paper specimen deforms under tension. If wrinkles occur on the paper specimen as a result of the tensile load, the measurement of paper properties such as in-plane Poisson ratio will be subject to substantial error [5].

Here, we apply theoretical and experimental approaches to the problem of tension buckling of paper materials and proceed to develop a systematic method for overcoming such wrinkle-related problems.

Tension Buckling Theory

In theory, all stresses in a rectangular specimen subjected to uniaxial tension should be zero except for those in the loading direction. Actually, however, lateral compressive stress builds up around the grips because of the interaction between grip restraint effects and the Poisson effect in the paper as it deforms. When tensile load is applied, the grips prevent lateral contraction of the specimen in the vicinity of the grips, which induces the development of lateral tensile stress. A reactive lateral compressive stress forms near the lateral tensile stress field toward the centre of the specimen and the combined tensile and compressive stresses form a bending couple.

The resultant lateral compressive stress may be very small compared to the axial tension, but is often great enough to cause the tension buckling phenomenon. The compressive stress across the width near the grips is not constant, but reaches a maximum at the centre of the width and approaches zero at the side edges. Thus the tension buckling starts at the centre of the width near the grips and propagates lengthwise along the specimen. The mode and severity of the buckling seem to depend on the specimen geometry, specimen mechanical properties, grip boundary conditions, and applied tensile loading.

Exact solutions for tension buckling behaviour near the grips have yet to be worked out. However, we know that the post-buckling shape of the specimen for areas that are distant from the grips is always cylindrical. If one applies cylindrical buckling theory to this region, the buckling stress should be proportional to the square of the ratio of specimen width to thickness [6]. Therefore, for a constant width, a thinner paper is much more prone to tension buckle than a thicker paper, other things equal.

Finite Element Approach

A finite element method (FEM) for an infinitesimal elastic analysis was used to generate the lateral compressive stress pattern that develops near the grips when the specimen is under uniaxial tensile loading. We used NASTRAN, and isoparametric linear quadrilateral elements in the analysis.

Figure 1(a) shows proportions for four different tensile specimen models shown in half lengths. Figures 1(b) and

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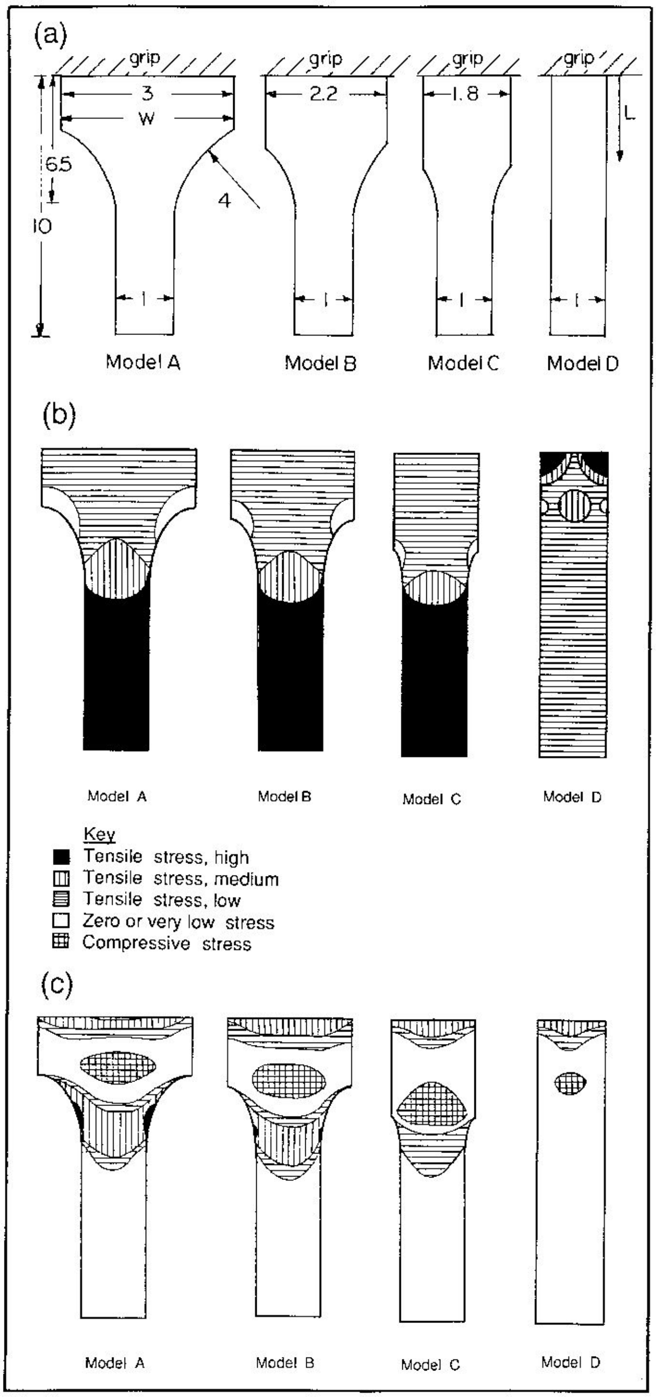

Fig. 1. (a) Dimensions of tensile specimen models; (b) stresses in the direction of loading (longitudinal axis); (c) stresses in the lateral direction.

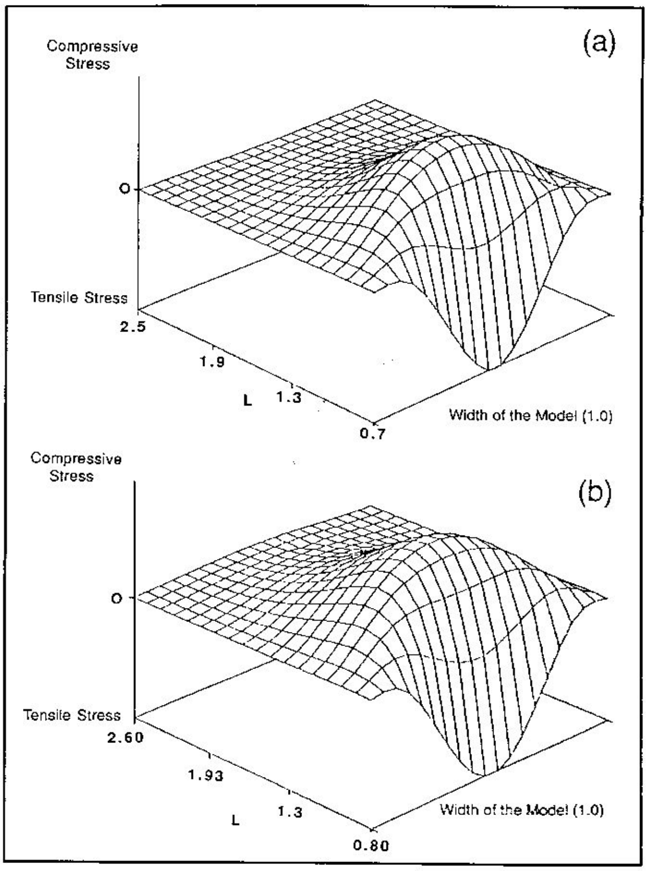

Fig. 2. (a) Lateral stress pattern for an isotropic model. L is an aspect ratio shown in Model D in Fig. 1(a).
$\nu_{xy} = 0.3$, $G_{xy}/E_{MD} = 0.375$;
(b) Lateral stress pattern for an orthotropic model. L is an aspect ratio shown in Model D in Fig. 1(a). The ratio of Young's moduli of the loading direction to the lateral direction in this model is 4.0.
$\nu_{xy} = 0.6$, $G_{xy}/E_{MD} = 0.162$.

1(c) show the axial loading direction stresses and the lateral stresses of models with isotropic elastic symmetry calculated from the FEM, respectively, when the models are subjected to 1% strain. We have also used models with orthotropic elastic symmetries in the FEM. The results yield very similar stress patterns to those from the isotropic models.

In Figure 1(b), the axial stresses (all tensile) are categorized into four different levels, as described in the legend. For necked-down models, the highest stresses are always located in the neck. For the rectangular model, a complex pattern of high and low tensile stresses arises in the vicinity of the grip (at the specimen corners next to the grip, stresses approach infinity according to the theory of elasticity). The stresses become quite uniform at locations further from the grip.

In Fig. 1(c), the lateral stresses are categorized into five different levels, one of which is compressive. The rectangular specimen model shows an oval of lateral compressive stresses near the grip, but no compensating tensile stresses exist below the compressive stress region. When the compressive stress near the grip goes beyond the critical limit, the specimen will buckle laterally; as axial tension on the specimen increases, the buckling will propagate axially along the length of the specimen. We present the lateral compressive patterns near the grips three-dimensionally for isotropic and orthotropic symmetry models of rectangular specimens in Fig. 2. Values for Poisson ratio (ν_{xy}) and the ratio of machine-direction Young's modulus to the in-plane shear modulus are given for this figure in the legends.

From Fig. 1(c), it is evident that there are also compressive stresses near the grips of necked-down samples. However, all the necked-down models have compensating tensile stresses below the compressive stress region. Furthermore, models A and B in Fig. 1(c) have their *maximum* tensile stresses below the compressive stress region. Therefore, even though there are lateral buckling forces near the grips, these tensile stresses will prevent the buckling from propagating, and will maintain flatness in the centre span of the specimen.

From this FEM analysis, we can expect buckling or wrinkling consequences from the lateral compressive stresses in terms of a change in shape of the speci-

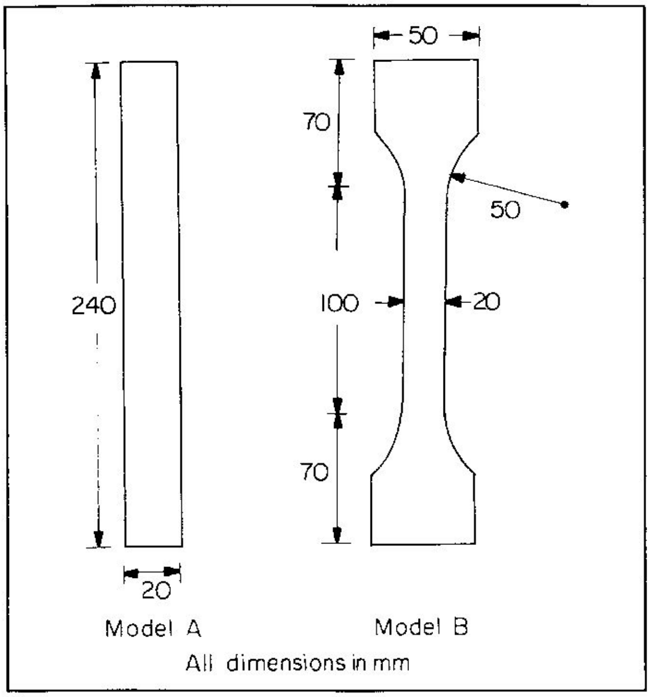

Fig. 3. Models of rectangular and necked-down specimens used for tension test trials. Basis weight: 25 g/m².

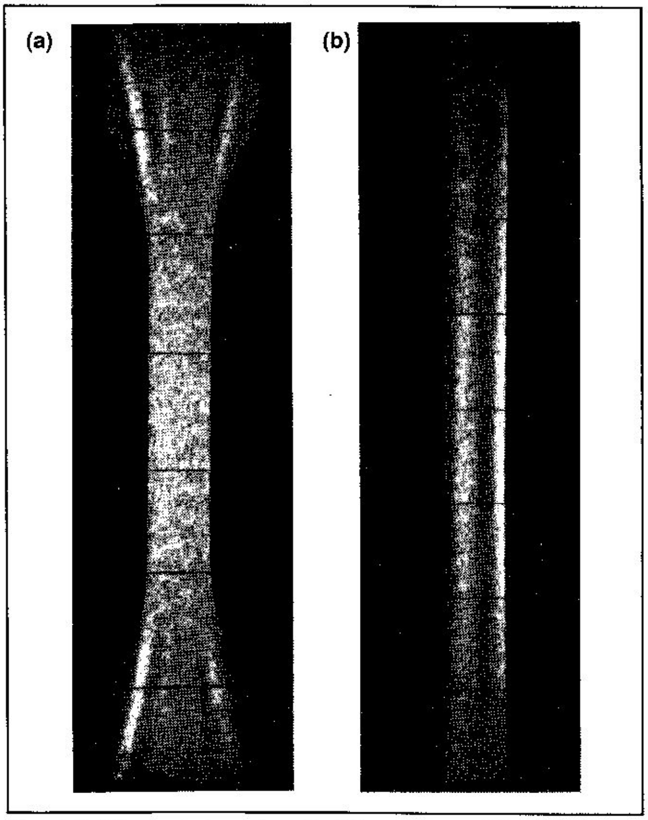

Fig. 4. (a) Necked-down tensile CD specimen (Model B in Fig. 3). Photographed at 6.5% of crosshead movement (close to total failure). The central section is wrinkle free. (b) Rectangular tensile CD specimen photographed at 3% of crosshead displacement (Model A in Fig. 3). Wrinkling occurs along the entire length.

men under tension. For a necked-down specimen, we expect buckling near the grips, but an absolutely flat shape starting a certain distance from the grips. For a thin rectangular specimen, we expect tension buckling all over the specimen as the test proceeds.

EXPERIMENTAL

Three kinds of low basis weight (25 g/m²) paper—tissue handsheets plus a machine-made tissue of two different fibre orientation distributions—were prepared in the form of necked-down specimens and a rectangular specimen (width 20 mm; length 240 mm). These specimens are shown in Fig. 3. During tensile tests, wrinkles developed on the specimens as shown in Fig. 4. As we predicted, a necked-down specimen will retain its flatness in the neck portion; wrinkles only appear in the wide regions near the grips. The flat area can be used for the accurate measurement of the deformations and local strains in the specimen. As loading progressed in a rectangular specimen such as shown in Fig. 4(b), wrinkles spread over the full area between the grips. We saw small-amplitude wrinkles even in a high basis weight paper (300 g/m²) at a strain level of 3-4% [2].

Each specimen under tensile load was photographed from an inclined angle in order to measure the wrinkles induced by tension buckling. The procedure is summarized in the Appendix. Figures 5 and 6 show the sheet surface topography at different strain levels of rectangular specimens in the machine direction (MD) and cross-machine direction (CD), respectively. Wrinkles have appeared at the 2% strain level in Figs. 5(b) and 6(b).

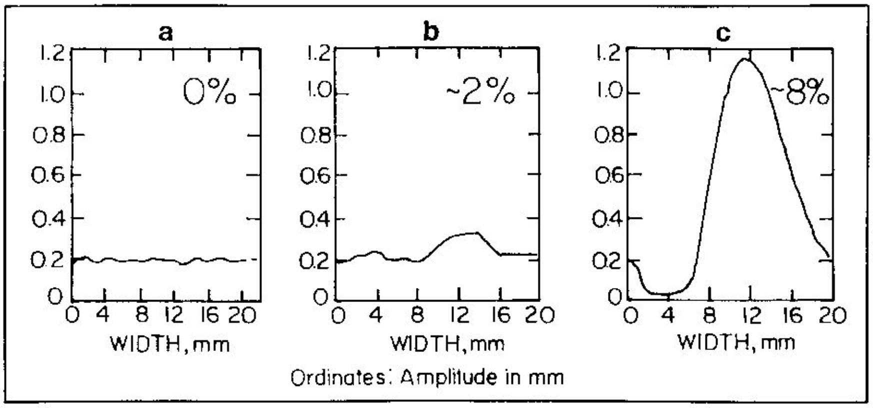

Fig. 5. Wrinkling of a rectangular tensile specimen in the machine direction. Specimen strain is shown at upper right of each graph.

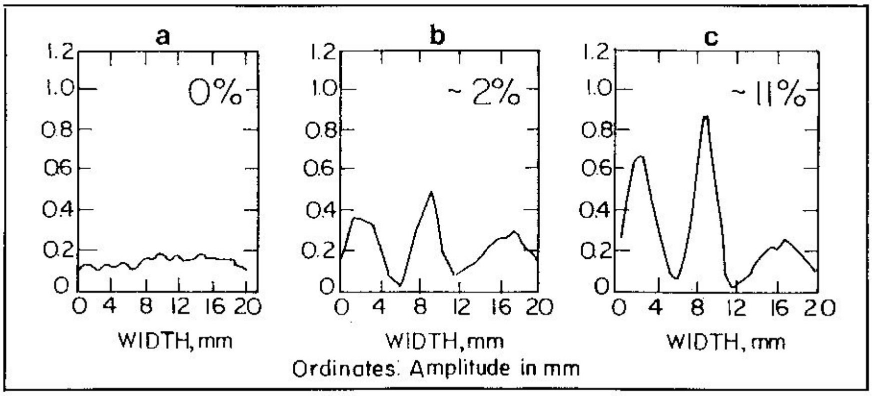

Fig. 6. Wrinkling of a rectangular tensile specimen in the cross-machine direction. Specimen strain is shown at upper right of each graph.

These wrinkles increase in amplitude until failure.

Figures 7 and 8 show the variation of sheet surface topography in the neck area of necked-down specimens at different strain levels in the MD and CD, respectively. Wrinkles do not appear until just before failure at very high strain levels. In fact, there is no wrinkling until the specimen reaches the start of tensile failure.

CONCLUSION

The uniaxial tensile test is a very important test for determining many mechanical properties of paper. By carrying out this test very carefully, we can obtain most of the in-plane and some of the out-of-plane mechanical properties. Wrinkles induced by tension buckling of the specimen should not be ignored when one wants to obtain extra information in addition to the tensile strength of the paper. The two-sidedness of paper could also account for some slight initial waviness of the sheet. A properly designed necked-down specimen allows perfectly flat deformation of the specimen to occur in the elastic as well as the inelastic range right up to failure.

Especially for light basis weight paper, necked-down specimens should be used in tensile testing. For the application of optical techniques to the measurement of in-plane deformation two-dimensionally, the prevention of wrinkle of the specimen is critical. Paper on a paper

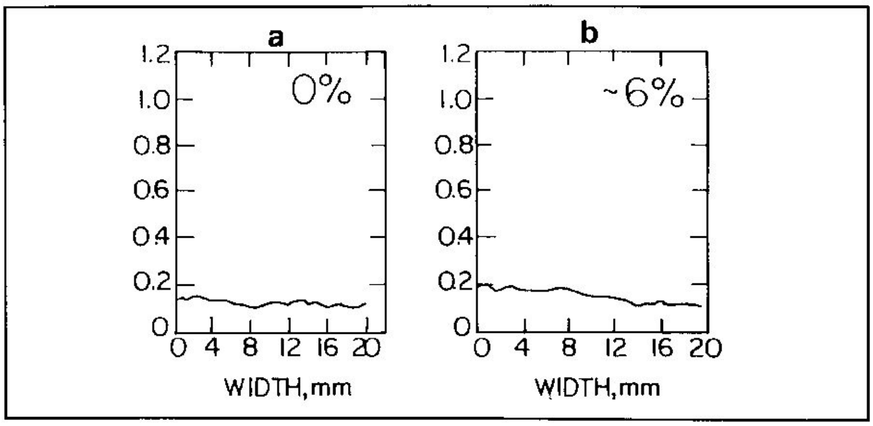

Fig. 7. Wrinkling of necked-down tensile specimen in the machine direction. Specimen strain is shown at upper right of each graph.

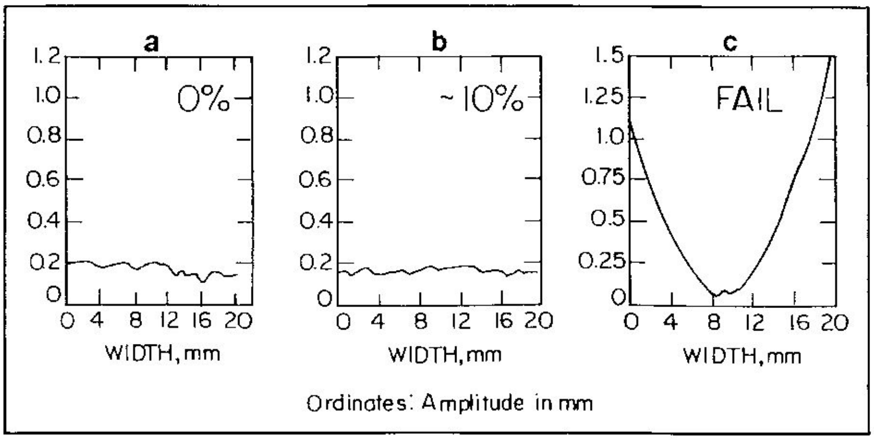

Fig. 8. Wrinkling of a necked-down tensile specimen in the cross-machine direction. Specimen strain is shown at upper right of each graph.

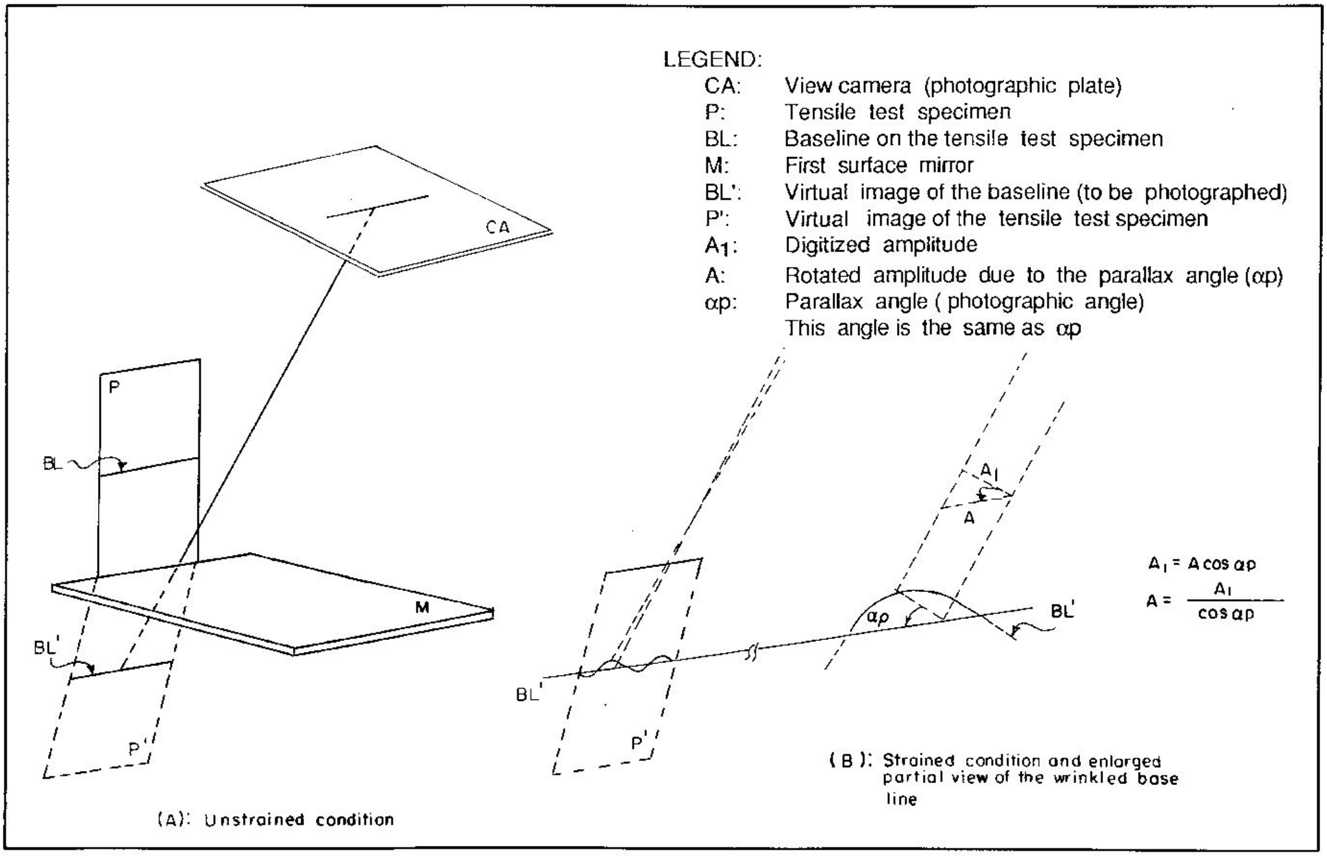

Fig. 9. Perspective view of method for examination of specimen flatness/wrinkling.

machine is usually under tensile stress, which often causes wrinkles and creases in the paper.

Our experiments show that a transition ratio between the width at the grips and the neck width of 2.5 or more is safe in most cases. However, if a special specimen geometry is used in the experiment, we recommend verification of the flatness of specimens in all strain ranges according to the procedure given in the Appendix.

APPENDIX

Wrinkle Determination

We draw a fine horizontal line at the centre of the free-span area of the specimen, and photograph the distortion of the line from the unstrained state to failure strain by observing it with a mirror at an inclined angle as shown in Fig. 9. The amplitudes of the wrinkles are calculated by comparing the digitized data from the unstrained state with those in various strained states, and by converting the data of differences to real wrinkle amplitudes according to the inclination angle of the camera (see equations in Fig. 9).

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ABSTRACT: When thin rectangular specimens of materials such as paper are tested in tension, wrinkles parallel to the loading direction often appear. The condition is termed "tension buckling". The propagation of these wrinkles can be confined to small areas near the grips if necked-down specimens are used. The authors use finite element analysis and mechanical and optical test methods to demonstrate the phenomenon, its magnitude, and methods to control it in this paper.

RÉSUMÉ: L'essai sous tension d'échantillons rectangulaires de matériaux minces tels que le papier engendre souvent la formation de rides dans le sens de la charge. Cette condition est qualifiée de "gondolage par tension". La propagation de ces rides peut se limiter à de petites surfaces près des points de préhension si l'on a recours à des échantillons de forme rétrécissante. Nous avons fait appel à l'analyse des éléments finis et aux méthodes d'essai optiques et mécaniques pour démontrer l'existence de ce phénomène, son importance et ses méthodes de contrôle.

KEYWORDS: BUCKLING, TENSILE TESTS, OPTICAL MEASUREMENT, WRINKLES, PARAMETERS, POISSON RATIO, ELASTIC STRENGTH, SHEAR STRENGTH, THEORIES, STRESS ANALYSIS.

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